

# The Emergence of Glider Guns in Cellular Automata found by Evolutionary Algorithms

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**Abstract.** We aim to construct an automatic system for the discovery of computationally universal cellular automata, spatially. *Glider* and *glider gun* structures are required for such systems.

In this paper, a large number of glider guns are presented which have spontaneously emerged through the use of a genetic algorithm. A classification of glider guns that takes into account the number of emitted gliders is also proposed.

## 1 Introduction

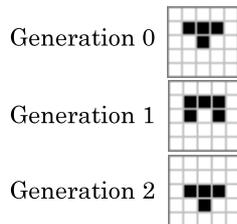
The emergence of computation in complex systems with simple components is a hot topic in the science of complexity [1]. A uniform framework to study emergent computation in complex systems are cellular automata (CA) [2]. CAs are discrete systems in which a population of cells evolves from generation to generation on the basis of local transitions rules [3].

Some automata have been shown to have the important property of computational universality, i.e., they are capable of simulating a Turing Machine [2]. The well-established problems of the universality in cellular automata have been tackled by a number of people in the last thirty years [4], [5], [6], [7], [8] and this remains a fruitful area where amazing phenomena at the edge of theoretical computer science and non-linear science can be discovered.

The most well known universal automaton is the Game of Life [9]. It was shown to be universal by Conway et al. in 1982 [10], who employed *gliders*, mobile self-localized patterns of non-resting states, and *glider guns*, i.e., a pattern which, when evolving alone, periodically recover their original shape after emitting some gliders.

Sapin et al [11, 12] discovered an universal CA which is universal because it can simulate the Game of Life. This was achieved by using two genetic algorithms (GAs) [13] in sequence searching for automata which exhibit gliders, named *R*, and then accepting a *glider gun* [14, 15]. This automaton is shown to be universal in [11, 16] thanks to the presence of the glider gun.

In this paper, inspired by genetic algorithms that searched for automata which exhibit gliders [14, 15], a GA that searches for glider guns is presented.



**Fig. 1.** Glider.

A large number of guns is found for the glider in figure 1. The guns are fully described using a new classification of glider guns.

The paper is arranged as follows: Section 2 describes previous related work. Section 3 sets out the characteristics of the evolutionary algorithms and the results are described in Section 4, along with the presentation of some interesting examples found by the GA. The last section summarizes the presented results and discusses directions for future research.

## 2 Previous work

### 2.1 Cellular automata

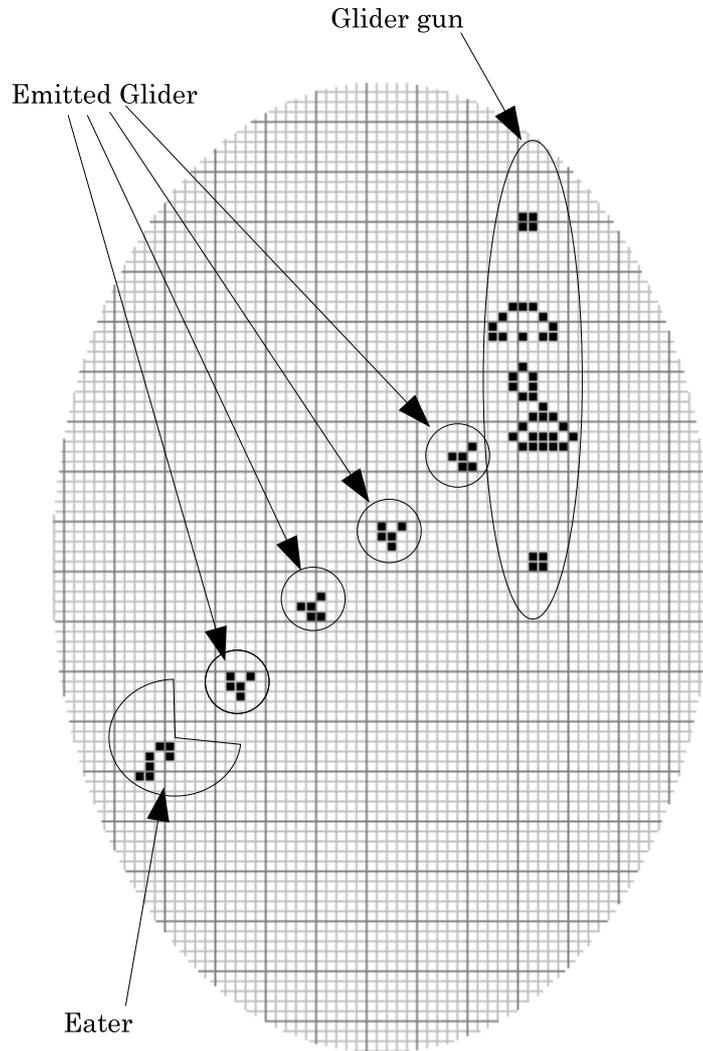
In 1970, Conway discovered a special automaton of  $\mathcal{I}$  (that he called the Game of Life) that was later popularized by Gardner in [9]. In [10], Conway, Berlekamp, and Guy show that the Game of Life can implement any function calculable by a Turing machine. Their proof of the universality of the Game of Life uses gliders, glider guns, and eaters. These patterns are shown in figure 2.

A glider gun is a pattern that emits a stream of gliders (used to carry information) and an eater destroys gliders. It is possible to combine glider guns and eaters in order to simulate logic gates and circuits. In [17], Rendell gives an explicit proof of the universality of the Game of Life by showing a direct simulation of counter machines.

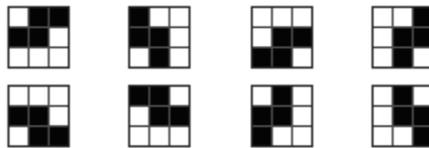
In the paper we constrain ourselves to 2D isotropic CA  $\mathcal{I}$ , with rectangular 8-cell neighbourhoods: if two cells have the same neighbourhood states by rotations and symmetries, then these two cells take the same state at the next generation [18]. Figure 3 shows neighbourhood states that are the same by rotations and symmetries.

There are 102 neighbourhood states that are different by rotations and symmetries, meaning that there are  $2^{102}$  different automata in  $\mathcal{I}$ . An automaton of this space, shown in figure 4, is described by telling what will become of a cell in the next generation, depending on its neighbours.

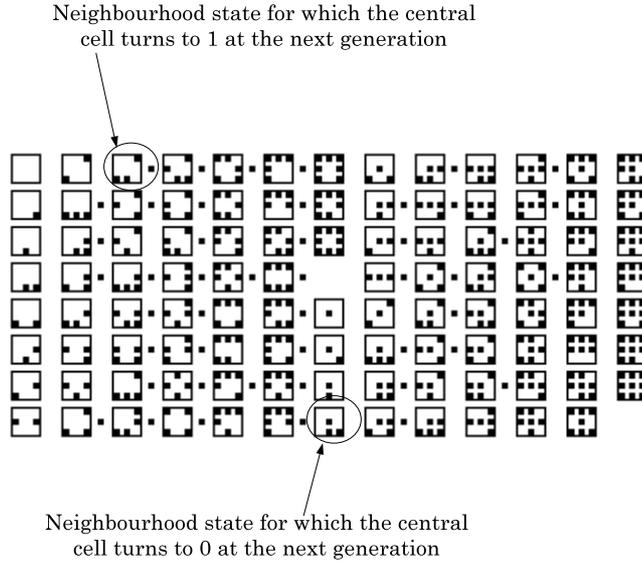
For conciseness and readability, the following convention has been adopted for the automaton presentation. Each neighbourhood state is referred to by a letter, shown figure 5, and the number of cells from 0 to 8 among it. In the convention, the neighbourhood states that allow the central cell to survive is noticed in the



**Fig. 2.** Glider, glider gun and eater of the Game of life.

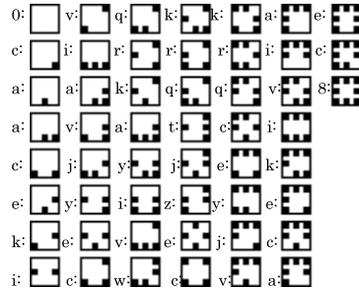


**Fig. 3.** neighborhood states that are the same by rotations and symmetries.



**Fig. 4.** The squares are the 102 neighbourhood states describing an automaton of  $\mathcal{I}$ . A black cell on the right of the neighbourhood state indicates a future central cell.

first part after the letter *s* of the automaton notation and the neighbourhood states that allow the central cell to born is notice in the second part after the letter *b*. For example, the notation *s2iv3a8/b4i8* means a cell survives for the neighbourhood states *2i*, *2v*, *3a* and *8*.



**Fig. 5.** Neighbourhood states.

## 2.2 Evolutionary algorithm

In order to search for another universal automata other than Life, we use an evolutionary algorithm [13], that incorporates aspects of natural selection or survival

of the fittest. It maintains a population of structures (usually initially generated at random) that evolves according to rules of selection, recombination, mutation, and survival referred to as genetic operators. A shared “environment” is used to determine the fitness or performance of each individual in the population. The fittest individuals are more likely to be selected for reproduction through recombination and mutation, in order to obtain potentially superior ones.

### 2.3 Evolving Cellular Automata

Previously, several good results from the evolution of cellular automaton rules to perform some useful tasks have been published. Mitchell et al. [19–22] have investigated the use of evolutionary computing to learn the rules of uniform one-dimensional, binary CAs. Here a Genetic Algorithm produces the entries in the update table used by each cell, candidate solutions being evaluated with regard to their degree of success for the given task — density and synchronization.

Sipper [23] has presented a related approach, which produces non-uniform solutions. Each cell of a one or two-dimensional CA is viewed as a GA population member, mating only with its lattice neighbours and receiving an individual fitness. He shows an increase in performance over Mitchell et al.’s work, exploiting the potential for spatial heterogeneity in the tasks. Koza et al. [24] have also repeated Mitchell et al.’s work, using Genetic Programming [25] to evolve update rules. They report similar results.

The manual search for gliders in cellular automata has been described by A. Wuensche who used his Z-parameter and entropy [26], A. Adamatzky et al. with a phenomenological search [27], D. Eppstein [28] or J. Heudin [29]. Only Sapin et al.’s work [16] has considered the emergence of glider-based universality in CAs via a GA.

### 2.4 R

As discussed above, in [14], in order to find computationally universal automaton other than the Game of Life, Sapin et al. used an evolutionary algorithm to look for automata in  $\mathcal{I}$  exhibiting gliders. Among all the found automata with gliders, some automata also show glider guns. An automaton  $R$ , shown in figure 6, where a gun was found was proven to be universal in [11, 12].

The idea of the proof is to simulate the Game of Life by  $R$ . The logic universality of  $R$  is shown by implementing a NAND gate as well as intersections and synchronizations of patterns. The value of a cell in the Game of Life at the current generation is the result of a logic formula that accepts in entry the values, at the previous generation, of the simulated cell and of its eight neighbours. The automaton  $R$  can implement any logic circuit so it can simulate a cell of the Game of Life. The simulation of the Game of Life is then shown by carrying out a tiling of a surface with the identical simulation of cells.

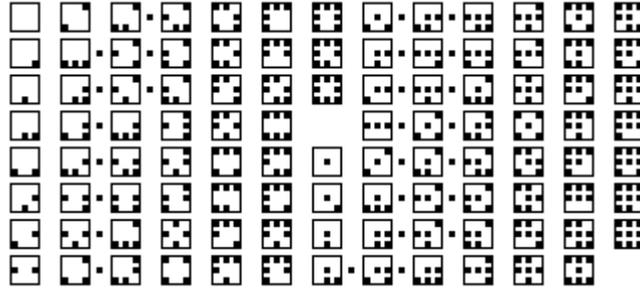


Fig. 6. Rule R.

### 3 Characteristic of the Evolutionary Algorithm

This section describes the parameters of the genetic algorithm that found the guns for the glider in figure 1. Several parameters had to be tried before the best result was found.

#### 3.1 Parameters

**Fitness Function** The computation of the fitness function is based on the one used in [14]. A random configuration of cells are evolved by the tested automaton. After this evolution, the presence of gliders  $G$  is checked by scanning the result of the configuration of the cells. The value of the fitness function is the number of gliders that appeared divided by the total number of cells.

**Initialization** The search space is the set  $\mathcal{I}$  described in Section 2. An automaton of this space can be described by determining what will become of a cell in the next generation, depending on its neighbours. An individual is an automaton coded as a bit string of 102 Booleans representing the values of a cell at the next generation for each neighbourhood state.

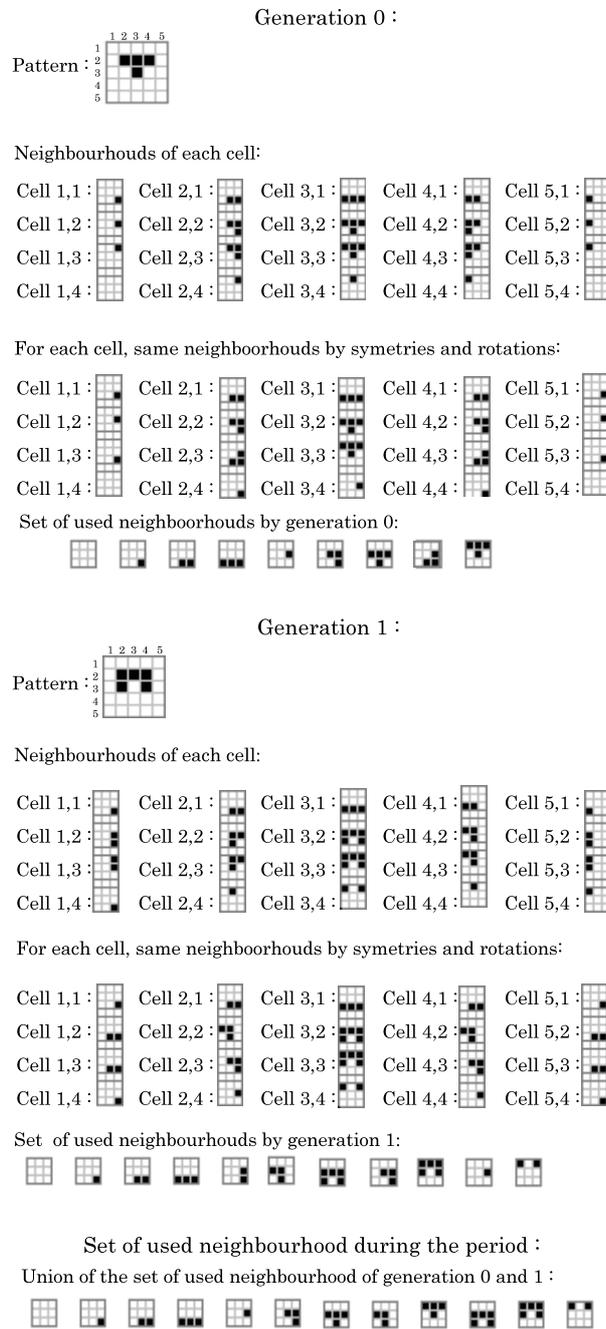
The 102 bits of an automaton are divided in two subsets. The first subset is the neighborhood states used by the glider  $G$  and their values are determined by the evolution of  $G$ . The process that determines these neighborhood states is detailed for the glider in figure 1 in figure 7.

The second subset are initialised at random.

**Genetic Operators** The mutation function simply consists of mutating the bits among the second subset of the 102 bits with a probability of 1 percent. Individuals are crossed over with a locus situated exactly on the middle of the genotype. The selection is a binary tournament of size 2.

**Evolution Engine** A non-elitist strategy in which the new population is made of only children is used.

**Stopping Criterion** The presence of a glider gun is continuously checked. The test is inspired by Bays' test [30] and also used in [15]. After the evolution of



**Fig. 7.** Detail of the construction of set of neighborhood state that are used by a glider.

the random configuration of cells, the pattern are isolated [15] and tested in an empty universe. If a pattern  $P$  reappears at the same place with gliders then the pattern  $P$  is a glider gun. When a glider gun is found the algorithm stops.

The value of the fitness function and the generation of the best rule are memorized. If after ten new generations the algorithm has not found a better rule the algorithm stops.

## 4 Results

The results of the genetic algorithm for the glider in figure 1 are described here.

971 different glider guns are found emitting this glider. In order to determine if a gun is new, the set of neighbourhood states used by the given gun are compared to the ones of the other guns. All the 971 guns can be found in [31] in Life format. The guns were classified by the number of streams emitted during their period.

### 4.1 One stream Guns

Two different types of streams can be emitted by a gun. If the period of the gun is a multiple of the period of the glider, the emitted gliders are in the same state during a generation of the automaton. If not the glider is not in the same state.

The glider in figure 1 is 2, so if the period of the gun is even, the gliders of the stream are in the same states. This kind of gun will be called Phased One Stream Gun or POS Gun and if the period is odd the gun is called Dephased One Stream Gun or DOS Gun. The two following subsections describe these two kinds of discovered guns.

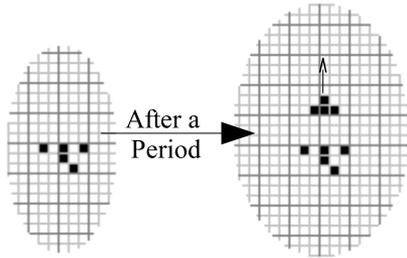
**POS Gun** Figures 8 shows a POS gun with a glider emitted in the same state. The period of this gun is 10.

**DOS Gun** Among the 971 discovered guns, there is only one DOS Gun. Figure 9 shows a gun with gliders emitted in the same state. The period of this gun is 7.

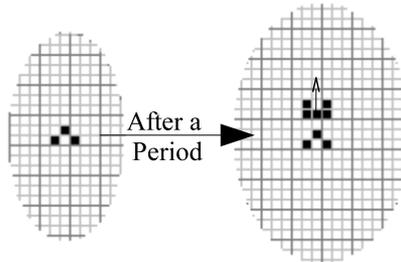
### 4.2 Two Stream Guns

Two streams emitted by a discovered gun can be orthogonal or parallel. If the streams are parallel then they can be lined up or not lined up. So, there can be three cases:

- Orthogonal.
- Lined up.
- Not lined up.

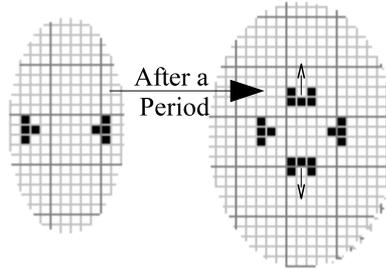


**Fig. 8.** A POS gun. This gun can be noticed:  
s01c2aki3ajec4aywkqjec5kyvi6a7ec8/b2civ3ae4aivrze5krcci6viec7ec8.

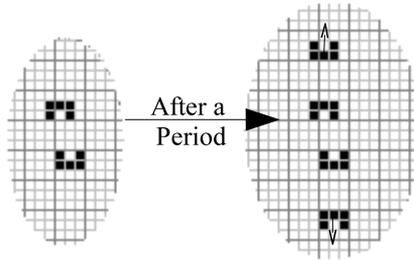


**Fig. 9.** A DOS gun. This gun can be noticed:  
s1c2av3yecr4zec5kqeyja6ke7c/b2ceki3aeqr4ykrqe5rqi6vea7c8.

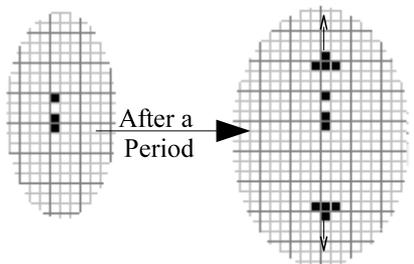




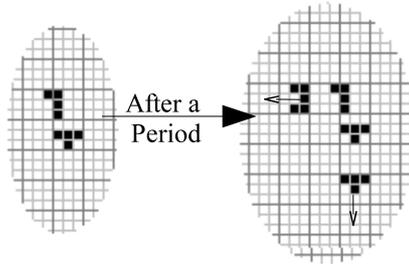
**Fig. 11.** SLT Gun. This gun can be noticed:  
[s2acki3avjyeq4vktjec5reyj7e8/b2ce3avjck4iwrtjzec5krqceyv6ea7c.](#)



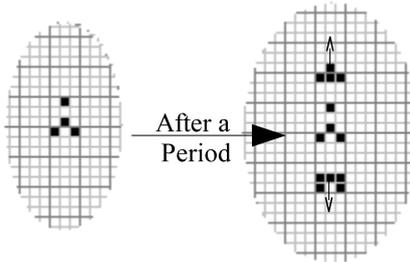
**Fig. 12.** SUT Gun. This gun can be noticed:  
[s1c2ac3avjeq4wrtjz5krceyv6eca7e8/b2ci3aerk4yiqj5qeyvai6vieca7e8.](#)



**Fig. 13.** PLT Gun. This gun can be noticed:  
[s1e2a3ayecqr4avrqtz5rqej6vec/b2ceiv3ak4arqe5rcvi6kec8.](#)



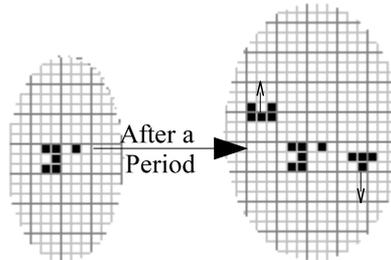
**Fig. 14.** DOT Gun. This gun can be noticed:  
s1c2av3vyek4avwrqtzec5rai6viec8/b2cki3aeqrk4yivwqzec5kejvi6ve7e.



**Fig. 15.** DLT Gun. This gun can be noticed:  
s1ce2ac3ayeqr4ayvre5krqcyai6ikc7c/b2cv3ae4yvwzec5kceyvai6ic7c.

**Dephased not lined up two-stream guns**, called DUT guns.

A period 18 DUT gun can be seen in figure 16.



**Fig. 16.** DUT Gun. This gun can be noticed: `s2a3vjyek4akrqjze5rqcvai6vec7c8/b2civ3aer4ayvwtje5eyvi6e`.

### 4.3 Four Stream Guns

The four streams emitted by a discovered gun can be directed towards the four cardinal directions or towards two cardinal directions. If the streams are directed towards the four cardinal directions then they can be lined up 2 by 2 or not lined up. So, there are three cases:

- Directed towards two cardinal directions referred to by the letter T.
- Directed towards four cardinal directions and lined up referred to by the letter L.
- Directed towards four cardinal directions and not lined up referred to by the letter U.

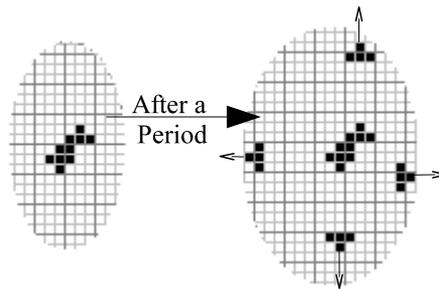
During the period of a discovered gun, four streams can be emitted at the same time during the period of the gun, at two different times or at four different times, the following terms are used respectively:

- Asynchronous referred to by the letter A.
- Semi-asynchronous referred to by the letter Y.
- Synchronous streams referred to by the letter S.

At a given generation, four streams can be in the same state or in two different states. The four streams are called phased streams referred to by the letter P or dephased streams referred to by the letter D.

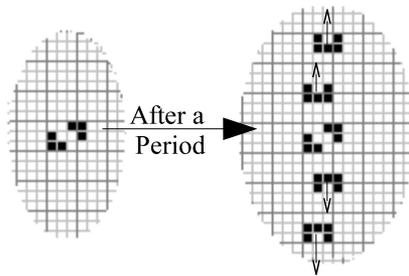
The discovered guns that emit four streams can be classified in eighteen categories:

- Phased, asynchronous and directed towards two directions four-stream guns, called PAT guns.  
Among the 971 discovered guns, there are no PAT guns.
- Phased, asynchronous and directed towards four directions lined up four-stream guns, called PAL guns.  
Among the discovered guns, no PAL guns appear.
- Phased, asynchronous and directed towards four directions not lined up four-stream guns, called PAU guns.  
Figure 17 shows a discovered period 16 PAU gun.



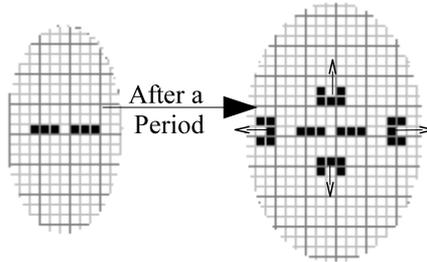
**Fig. 17.** PAU Gun. This gun can be noticed:  
s1c2acki3e4vkrqtje5krCY6vike7ec8/b2civ3ae4ywkqze5kcyvi6vike7ec8.

- Phased, semi-asynchronous and directed towards two directions four-stream guns, called PYT guns.  
A period 20 PYT gun is shown in figure 18.



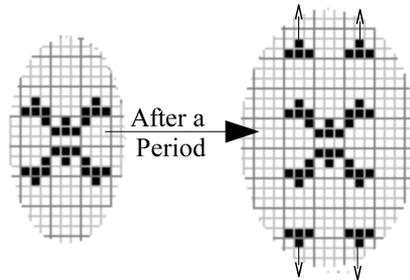
**Fig. 18.** PYT Gun. This gun can be noticed:  
s01ce2av3aeq4awkqzc5cea6vike7ec8/b2ci3ayeck4ayqtjz5kqyj6ike8.

- Phased, semi-asynchronous and directed towards four directions lined up four-stream guns, called PYL guns.  
Figure 19 shows a discovered period 12 PYL gun.



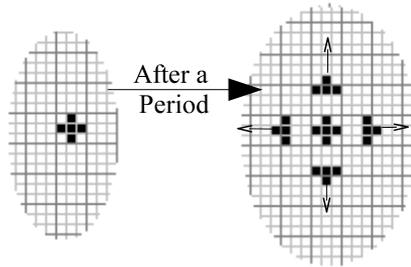
**Fig. 19.** PYL Gun. This gun can be noticed: `s1ce2ack3ae4avrqe5kqcvai6ec8/b2ce3ayecrk4yiqe5rqeai6ka7e`.

- Phased, semi-asynchronous and directed towards four directions not lined up four-stream guns, called PYU guns.  
Among the 971 discovered guns, there are no PYU guns.
- Phased, synchronous and directed towards two directions four-stream guns, called PST guns.  
Figure 20 shows a discovered period 10 PST gun.



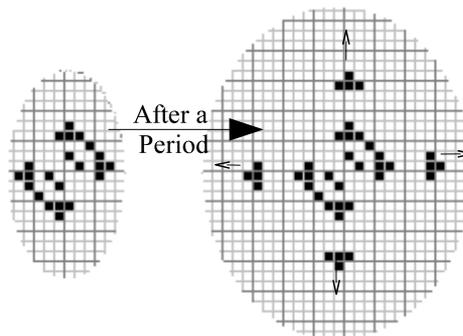
**Fig. 20.** PST Gun. This gun can be noticed: `s1c2aiv3avjerk4krtjz5rqcva7c/b2cki3ayecrk4yivqjc5kjvai6e7e8`.

- Phased, synchronous and directed towards four directions lined up four-stream guns, called PSL guns.  
A PSL gun is shown in figure 21.



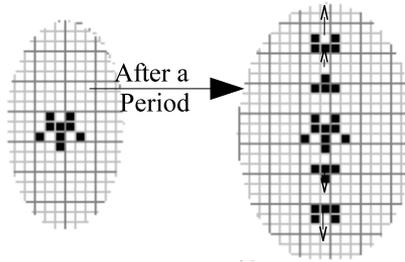
**Fig. 21.** PSL Gun. This gun can be noticed:  
[s1c2ai3ajyecqr4yvkrjtjec5kejv/b2cki3aeq4yivwkrqtzec5kei6ike](#).

- Phased, synchronous and directed towards four directions not lined up four-stream guns, called PSU guns.  
 Figure 22 shows a discovered period 10 PSU gun.

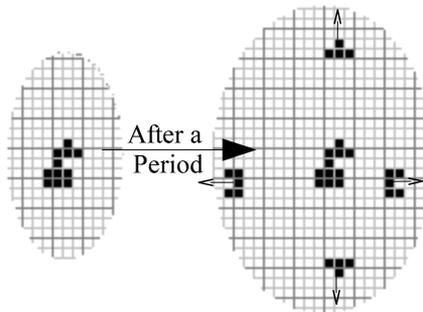


**Fig. 22.** PSU Gun. This gun can be noticed:  
[s1c2ack3er4vrtzc5kceji6vi7e8/b2civ3ayec4yqtjec5qceyi6ea7e](#).

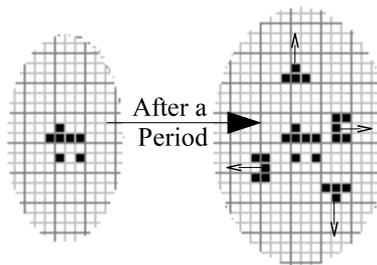
- Dephased, asynchronous and directed towards two directions four-stream guns, called DAT guns.  
 The discovered DAT gun in figure 23 have a period of 16.
- Dephased, asynchronous and directed towards four directions lined up four-stream guns, called DAL guns.  
 Figure 24 shows a discovered DAL gun of period 20.
- Dephased, asynchronous and directed towards four directions not lined up four-stream guns, called DAU guns.  
 One of the discovered DAU gun of period 12 can be seen in figure 25.
- Dephased, semi-asynchronous and directed towards two directions four-stream guns, called DYT guns.



**Fig. 23.** DAT Gun. This gun can be noticed:  
 s1c2ai3aek4vwkrqtze5krqcey6vkec7c/b2ckiv3ae4wjc5qcei6vik.

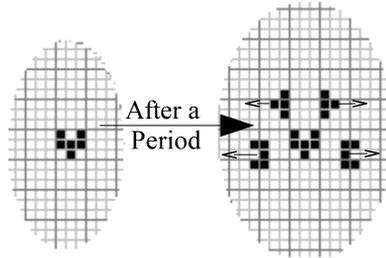


**Fig. 24.** DAL Gun. This gun can be noticed:  
 s02aiv3avjyecqr4wrtjc5ke6vea7e/b2cei3ayr4artzc5rqai6vikec7e8.



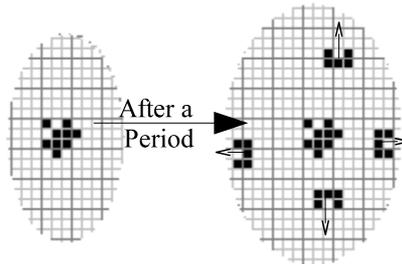
**Fig. 25.** DAU Gun. This gun can be noticed:  
 s01c2acv3ajyecqk4yvtj5krqy6eca7e/b2cei3acr4aikrjec5kcjvi6eca8.

Figure 26 shows a discovered DYT gun of period 10.



**Fig. 26.** DYT Gun. This gun can be noticed: [s1c2ak3avyeq4ywkzecz5rcyj6ea7ec8/b2ceiv3aje4yvqe5kceji7e8](#).

- Dephased, semi-asynchronous and directed towards four directions lined up four-stream guns, called DYU guns.  
Among the 971 discovered guns, there are no DYU guns.
- Dephased, semi-asynchronous and directed towards four directions not lined up four-stream guns, called DYU guns.  
Figure 27 shows a discovered DYU gun of period 16.



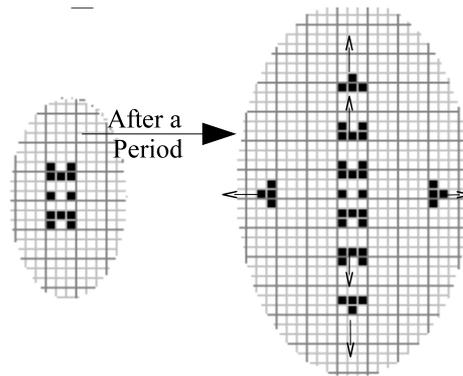
**Fig. 27.** DYU Gun. This gun can be noticed: [s1c2aiv3eq4wtjec5krqcy6ik8/b2ci3avyeq4ayiktjz5qcyi6ia](#).

- Dephased, synchronous and directed towards two directions four-stream guns, called DST guns.  
Among the 971 discovered guns, there are no DST Gun.
- Dephased, synchronous and directed towards four directions lined up four-stream guns, called DSL guns.  
There are no discovered DST guns.
- Dephased, synchronous and directed towards four directions not lined up four-stream guns, called DSU guns.  
No DSU guns appear.

#### 4.4 Six Stream Guns

Guns that emit six streams during their period are not very common among the 971 discovered guns. There are only two such types of gun. These kinds of guns are called 6 stream guns.

The first 6 stream gun, shown in figure 28, emits two gliders at generation 3, 8 and 14 and its period is 17.



**Fig. 28.** Discovered gun that emits six streams during its period. This gun can be noticed: s2aki3vjeq4ayvkqtje5rgev6viec7c/b2ceki3ar4yiec5qjai6ie7e.

The second 6 stream gun, shown in figure 29, emits two gliders at generation 12, 16 and 19 and its period is 20.

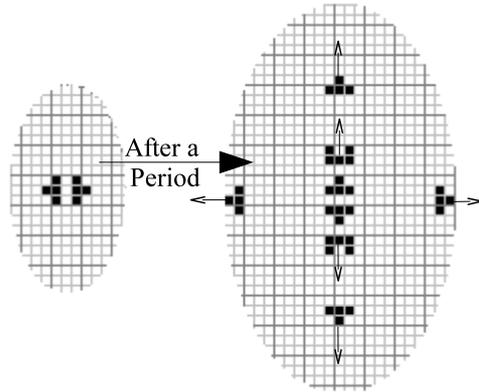
#### 4.5 Eight Stream Guns

Only one gun exists that emits eight streams among the 971 discovered guns. This gun is shown in figure 30

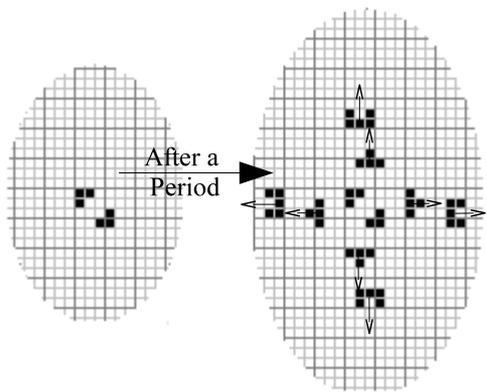
### 5 Synthesis and perspectives

This paper deals with the emergence of computation in complex systems with local interactions. An evolutionary algorithm searching for glider guns has been presented, building on previous work in [14, 15].

The algorithm succeeded in finding 971 glider guns [31] emitting the glider in figure 1. Only one gun was known of before for this glider, found by Sapin et al. The discovery of the emergence and existence of so many different glider guns for the same glider has only been possible through the use of the GA. Thus the results reported here represent a significant contribution to the area of complex systems that considers computational theory.



**Fig. 29.** Discovered gun that emits six streams during its period. This gun can be noticed: `s1ce2ak3jecqr4ave5krceyjvi6iea7ec/b2ckiv3ayer4yikqc5rcejvi6vc`.



**Fig. 30.** Discovered gun that emits eight streams during its period. This gun can be noticed: `s1ce2ac3eq4ayvrtjzec5krqcvjv6a8/b2cv3aye4yivwkrz5qcei6kc7ec8`.

All discovered guns are exhibited by certain numbers of automata. All these automata are potential candidates for being universal automata. Further goals are to determine this number of automata and to determine which automata are universal thanks to an automatic system for the demonstration of universal automata. Additionally, another domain that seems worth exploring is how this approach could be extended to automata with more than 2 states.

Future work could also evaluate all found rules and calculate for each rule some rule-based parameters, e.g., Langton's lambda [32]. All rules supporting glider guns may have similar values for these parameters that could lead to an answer to the question 'Where are the edges of computational universality?' and may therefore lead to a better understanding of the emergence of computation in complex systems with local interactions.

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